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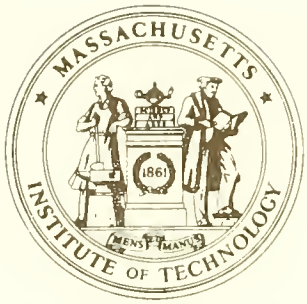

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RESOURCE EXTRACTION AND RECYCLING WITH ENVIRONMENTAL COSTS.*

Michael Hoel

Working Paper Number 197 April 1977

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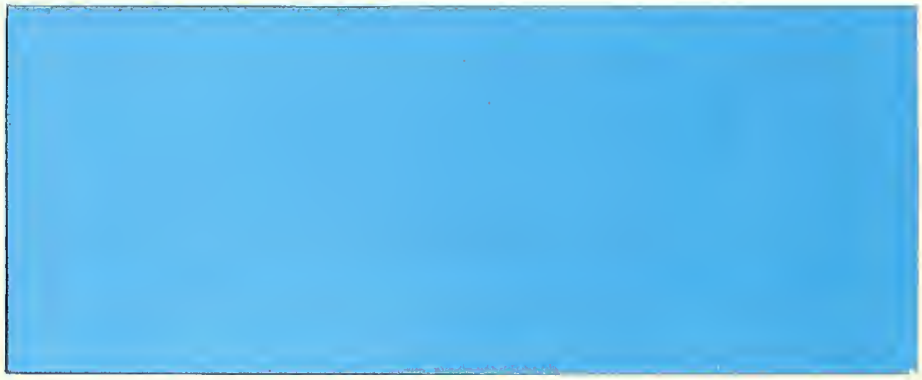

RESOURCE EXTRACTION AND RECYCLING WITH ENVIRONMENTAL COSTS.*

Michael Hoel

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Resource Extraction and Recycling with Environmental Costs.

By

Michael Hoel

1. Introduction.

Most of the literature treating the economics of exhaustible resources disregards the connection between resource extraction and the disposal of residuals in the environment. However, unless the products made from the resources are completely recycled once they have served their "final" good function, the resources will reappear as residuals in the environment. As long as these residuals are regarded as harmless, an analysis treating resource extraction may as well disregard the disposal of them. However, many residuals are harmful, or will become harmful once their stock is sufficiently large. Examples of harmful residuals are carbon dioxide (which can affect the climate), thermal pollution from all types of energy based or exhaustible resources, several toxic metals such as cadmium, lead, mercury and various radioactive elements, and other elements such as sulphur, phosphor, etc.

In addition to the types of environmental pollution mentioned above, the extracting activity itself may have environmental effects. Two typical examples are ecological consequences of oil-spills from off-shore oil derricks, and recreational and agricultural losses from strip mining.

Some recent literature has treated various aspects of resource extraction taking explicit account of environmental effects. d'Arge and Kogiku [2, 1973], Mäler [13, 1974, ch. 3] and Lusky [12, 1975] treat macroeconomic optimization problems where exhaustible resources as well

as environmental costs enter the analysis. Smith [20, 1972] focuses on the connection between recycling and waste accumulation, but only little attention is given to how the environmental costs affect resource extraction. More explicit attention to the consequences of environmental costs for resource extraction in a microeconomic context has been given by Schulze [18, 1974]. As we shall see, this part of his analysis will be similar to the special case we treat in section 4. Schulze also analyzes some consequences for resource extraction of possibilities of recycling, but disregards environmental costs in this part of his work. Finally, a more empirically oriented work in this field which should be mentioned is by Nordhaus [15 and 16, 1976]. Here the consequences of carbon dioxide pollution for the optimal use of energy resources are studied.

The present paper is a microeconomic partial analysis of how environmental costs affect optimal resource extraction. One important distinction from the work of Schulze [18, 1974] is that the present analysis includes either the possibility of recycling or a "backstop technology" capable of producing a perfect substitute for the resource. In particular, we shall investigate how the cost of recycling or producing the substitute will affect the resource extraction and the development of harmful residuals or other negative impacts on the environment.

2. Assumptions underlying the analysis.

The model we shall use in the following analysis will cover both types of environmental effects mentioned in section 1, i.e., both disposal of harmful residuals and harmful effects from the extraction process itself. Let us start by discussing the first of these effects together with how we shall treat recycling.

Denote the unextracted resource stock by $S(t)$, the stock of harmful residuals by $R_1(t)$ and the stock of harmless residuals by $Q(t)$.[1] From the law of conservation of mass[2] we must have

$$S(t) + R_1(t) + Q(t) = K \ , \tag{1}$$

where K is a constant.[3] Denoting the rate of resource extraction $x(t)$, we therefore have

$$\dot{R}_1(t) + \dot{Q}(t) = -\dot{S}(t) = x(t) \ . \tag{2}$$

We now assume that after having served its purpose embodied in a final good, the resource is disposed in the environment such that a share α (where $0 < \alpha \leq 1$) is a harmful residual (due to where and in what form this fraction is disposed) while the rest is a harmless residual. In the absence of recycling, the harmful residual therefore grows according to

$$\dot{R}_1(t) = \alpha x(t) - \delta R_1(t) \ . \tag{3}$$

Here $\delta \geq 0$ is a depreciation factor, representing the modification of residuals (from R_1 to Q) due to the assimilative capability of the natural environment. The same simplified representation has been used by Keeler, Spence and Zeckhauser [9, 1972], Plourde [17, 1972], Smith [20, 1972] and Strøm [21, 1973]. d'Arge and Kogiku [2, 1973], Lusky [12, 1975]

and Schulze [18, 1974] implicitly assume that $\delta = 0$. We shall treat this case as a special version of our more general model.

In addition to the depreciation process by the natural environment, the stock and flow of harmful residuals can be reduced by various types of pollution abatement activities. Such activities are included in the work of d'Arge [1, 1971], Keeler, Spence and Zeckhauser [9, 1972], Plourde [17, 1972] and Strøm [21, 1973]. In the present study we shall disregard these types of activities. However, the stock and flow of harmful residuals may also be affected by recycling activities, which will be included in our analysis. Assume that each unit of recycled material is based on a fraction β of harmful residuals and $1-\beta$ of harmless residuals. It seems reasonable to expect $\beta \leq \alpha$, as it usually is easier to recycle from waste in solid form than from waste in liquid or gas form, which often is the most harmful form of residual. Throughout our analysis, we shall assume that $0 < \beta \leq \alpha$, and as an important special case we shall consider $\beta = \alpha$. This special case can for instance arise if the material is collected for recycling before any of it is disposed of in the environment.

After the recycled material has served its purpose as a "final" good, it of course ends up as a residual in the same way as the virgin resource does. Disregarding the time between the date of recycling and the date of disposal our relation (3) therefore is changed in the following way when material is recycled at the rate $y(t)$:

$$(3') \qquad \dot{R}_1 = \alpha(x+y) - \beta y - \delta R_1 = \alpha[x + (1 - \frac{\beta}{\alpha})y] - \delta R_1 \ ,$$

where time references have been omitted for notational convenience, as they will be in most of the following formulas. We now define $a = 1 - \beta/\alpha$

and $R = R_1/\alpha$, thus changing the measuring unit of the harmful residual, (3') can then be rewritten as

$$\dot{R} = x + ay - \delta R \,, \tag{4}$$

where $0 \leqq a < 1$ due to our assumption $0 < \beta \leqq \alpha$.

We shall assume that the unit cost of recycling is a positive constant c. Furthermore, we shall disregard any constraint on how large the rate of recycling can be. The reason why such a constraint can exist is that if the duration of the material embodied in a final good is taken into consideration, there will be a limit at any time on how much material is available to recycle into additional goods. Another way of saying this is that the stock of material embodied in durable final goods cannot exceed the total amount of material available. However, such a limit can be disregarded as long as the initial stock of material is high compared with the long-run stationary level of recycling which we shall derive. If we were to consider a growing demand for the recycled material such a constraint would sooner or later become relevant, cf. Weinstein and Zeckhauser [22, 1974].

It should be noted that our specification of the recycling activity makes stationary states of y feasible (i.e., without R or Q ever becoming zero) as long as Q(0) is sufficiently large and we do not have the situation $\delta = 0$, $a > 0$. In this sense we have 100 per cent recycling. If $\delta = 0$ and $a > 0$ any constant level of y will sooner or later violate $Q(t) \geqq 0$, i.e., a constant y is impossible in this case. This case is characterized by some of the materials getting lost in the sense that they forever (since $\delta = 0$) end up as harmful materials, while recycling requires a larger fraction of harmless residuals than the fraction of harmless

residuals in the waste ($\beta < \alpha$, or $a > 0$). For instance, if some material is partly disposed of in gas form in the atmosphere and recycling only is possible from solid waste, a constant level of recycling will only be possible for a finite time period. A more thorough treatment of such cases is given by Schulze [18, 1974], where losses of material to unrecoverable forms of residuals occur both in the use of the material and from the stock of recoverable residuals. In the following analysis we shall disregard such cases, i.e., we shall not consider the case $\delta = 0$, $a > 0$.

It is clear that the way we have introduced the recycling activity, $y(t)$ could as well be interpreted as a substitute produced by a "backstop technology". (See Nordhaus [14, 1973], Dasgupta and Stiglitz [4, 1976] and Hoel [5, 1976] and [6 and 7, 1977] for studies of resource extraction with such a backstop technology but without environmental costs.) In this case we would have $a = 0$ in the case of a "clean" substitute for the resource (for instance solar energy for fossil fuels). If the substitute also had some negative environmental effect (for instance nuclear energy), we would have $a > 0$.[4)] It is of no practical importance whether one interprets y as recycled material or as a substitute, we shall stick to the recycling interpretation in verbal discussions in the following.

We mentioned earlier that the negative environmental effect could also be thought of as a direct consequence of extraction activities, such as strip mining and oil-spills. In this case we can have complete irreversibiliy (i.e., $\delta = 0$), for instance when recreational and aesthetic values associated with undisturbed environments are lost (see Krutilla and Fisher [11, 1975, ch. 3] for a discussion of whether or not there

is full irreversibility in such cases). We can also have $\delta > 0$ in this case. An example of this would be oil-spills, where the oil gradually gets degraded by the sea. Like before, y can be thought of either as recycled material or as a produced substitute. In both bases we get $a > 0$ or $a = 0$ depending on whether or not there are any negative environmental effects from the recycling or production activity.

Although the case $\delta = 0$, $a > 0$ excluded the possibility of a constant y forever in the case of harmful residuals and recycling, this is not true if either the environmental effects are a consequence of the extraction activity itself or if y is a substitute produced by a backstop technology. In these two cases there is nothing to prevent y constant forever even if $\delta = 0$, $a > 0$.

The society's welfare function is assumed to be given by

$$(5) \qquad W = \int_0^\infty e^{-rt}[U(x+y) - E(R) - cy]dt \ ,$$

Here r is an exogeneously given positive discount rate, assumed constant for simplicity. $U(x+y)$ is the gross utility for society at any instant of time derived from having the rate of extraction plus recycling equal to $x+y$. We shall assume that $U' > 0$, $U'' < 0$, $c/(1-a) < U'(0) < \infty$ and $\lim_{y\to\infty} U'(y) = 0$. $E(R)$ is the disutility for the society at any instant of time from having a stock of harmful residuals equal to R. We shall assume that $E(R)$ is convex, $E'(R) > 0$ and $E'(R_o)/(r+\delta) < c/(1-a)$, where R_o is the initial stock of harmful residuals. This last inequality will be seen to be necessary in order to have any resource extraction take place in an optimal solution. Except for the special case in section 6, we shall also assume that $E''(R) > 0$ and that $aE'(R)/(r+\delta) > U'(0)$ for a

sufficiently large value of R. Notice that the environmental effect given by E(R) only includes the stock of harmful residuals, and not the gross or net increase in these residuals. The term cy takes care of the cost of recycling y units of material, measured in the same units as the utility of extraction plus recycling and disutility of harmful residuals. To focus on the existence of environmental costs, we have assumed that traditional extraction costs are zero.[5)]

3. The general optimization problem.

The society's optimization problem is to maximize (5) subject to (4) and the initial condition $R(0) = R_o$ as well as

(6) $\quad \dot{S} = -x \ , \ S(0) = S_o \ ,$

and the nonnegativity constraints $x(t), y(t), S(t) \geqq 0$. The differential equation (6) takes care of the finiteness of the natural resource.

The current value Hamiltonian, written so that the shadow prices $\lambda(t)$ and $\mu(t)$ will be nonnegative, is

$$H = U(x+y) - E(R) - cy - \lambda(x + ay - \delta R) - \mu x \ .$$

The necessary conditions for an optimal solution imply that[6)]

(7) $\quad \dot{\lambda} = (r+\delta)\lambda - E'(R) \ ,$

(8) $\quad \dot{\mu} = r\mu \ ,$

(9) $\quad U'(x+y) \leqq \lambda + \mu \ , \text{ where } < \text{ implies } x = 0 \ ,$

(10) $\quad U'(x+y) \leqq c + a\lambda \ , \text{ where } < \text{ implies } y = 0 \ .$

Equations (9) and (10) define x and y as functions of λ and μ (and of course c and a). Inserting these functions into (4) and (6) gives us two differential equations in R, S, λ and μ. Together with (7) and (8) this gives us four differential equations in these four variables. We have two initial conditions ($R(0) = R_o$ and $S(0) = S_o$), in addition we must have

(11) $\quad \lim_{t\to\infty} S(t) \geqq 0 \ , \text{ where } > \text{ implies } \mu(t) \equiv 0$

and

(12) $\quad \lim_{t\to\infty} e^{-rt}\lambda(t) = 0 \ ,$[7)]

which together with our four differential equations and two initial conditions gives the time paths for R, S, λ and μ, after which the optimal paths for x and y can be found.

Let us look at some properties of our solution. First, let us look for a long-run stationary solution where $S = x = 0$ and where R and y are constant, denoted by R* and y*. From (10) we see that a constant positive y implies that λ is constant (and (12) is therefore satisfied), so that (7) and (10) give us

$$U'(y^*) = c + a\frac{E'(R^*)}{r+\delta} \quad . \tag{13}$$

When $x = 0$, (4) and $\dot{R} = 0$ give us

$$y^* = \frac{\delta}{a} R^* \tag{14}$$

when $a > 0$. Inserting this into (13) gives the long-run stock of the harmful residuals. Notice that when $a > 0$ and $\delta = 0$ we get $y^* = 0$ as long as $aE'(R)/(r+\delta) > U'(0)$ for R sufficiently large. In other words, even in cases where a constant y is physically possible when $a > 0$ and $\delta = 0$ (cf. p. 7), it is optimal to have $y^* = 0$.

When $a > 0$ and $\delta > 0$ so that (R^*,y^*) follows from (13) and (14), implicit derivation gives us the following results:

$$\begin{gathered} \partial R^*/\partial c < 0 \ , \ \partial R^*/\partial a \gtreqless 0 \ , \ \partial R^*/\partial \delta \gtreqless 0 \ , \ \partial R^*/\partial r > 0 \ , \\ \partial y^*/\partial c < 0 \ , \ \partial y^*/\partial a < 0 \ , \ \partial y^*/\partial \delta > 0 \ , \ \partial y^*/\partial r > 0 \ . \end{gathered} \tag{15}$$

It is rather surprising that the long-run stock of harmful residuals is higher the lower the cost of recycling is. The reason is that the lower

this cost is, the higher is the opportunity cost (measured by $(U'(y) - c)/a$) of refraining from adding an additional unit to the stock of harmful residuals. We may also find that the optimal long-run stock of harmful residuals is larger the larger the assimilative capacity of the environment (measured by δ) is. The reason for this is similar to the reason for $\partial R^*/\partial c < 0$. The other results speak for themselves, and need no comment.

When $a = 0$, (13) becomes

$$U'(y^*) = c \ , \tag{13'}$$

i.e., y^* is independent of a and δ. In this case we either have R^* constant and determined by earlier resource extraction (for $\delta = 0$), or R will decline towards zero (for $\delta > 0$).

Let us now look at how the resource extraction takes place in a period when there is no recycling. In this case we have $y = 0$ and (9) holds with equality. In other words, the marginal utility of using the resource at any time equals the sum of the two shadow prices $\lambda(t)$ and $\mu(t)$, which are the (undiscounted) shadow prices of reducing the stock of harmful residuals by one unit and of increasing the resource stock by one unit, respectively. Solving the differential equations (7) and (8) and using (12) gives us

$$\lambda(t) = e^{(r+\delta)t} \int_t^{\infty} E'(R(\tau))e^{-(r+\delta)\tau} d\tau \tag{16}$$

and

$$\mu(t) = e^{rt}\mu(0) \ , \tag{17}$$

where $\mu(0)$ is determined so that (11) holds. It should be noted that (16) and (17) hold for all t, and not only when $y = 0$. Since $E(R)$ is convex, a consequence of (16) is

$$(18) \qquad \frac{E'(0)}{r+\delta} \leq \lambda(t) \leq \frac{E'(R^m)}{r+\delta} ,$$

where R^m is the highest value R has along the optimal path.

From (7), (8) and (9) it is easy to see that when $y = 0$ and $x > 0$ we must have

$$(19) \qquad \frac{d}{dt}\,[U'(x)] = rU'(x) - [E'(R) - \delta\lambda] .$$

Without any environmental effects, i.e., $E'(R(t)) = 0$ for all t, it follows from (16) and (19) that we get the standard Hotelling [8, 1931] result, i.e., that the undiscounted marginal utility of using the resource should increase with a rate equal to the discount factor. When resource extraction affects the environment in a negative way, this rule is altered according to equation (19), where λ may be inserted from (16).

Before turning to some special cases, let us consider briefly if and when resource extraction and recycling may take place simultaneously. For this to occur, both (9) and (10) must hold with equality, i.e.,

$$(20) \qquad (1-a)\,\lambda(t) + \mu(t) = c .$$

Differentiating this equation and using (7) and (8) gives us

$$(r+\delta)(1-a)\,\lambda(t) - (1-a)\,E'(R) + r\mu(t) = 0 ,$$

which together with (20) gives

$$(21) \qquad (r+\delta)c - \delta\mu(t) - (1-a)\,E'(R) = 0 .$$

If $\mu(t) = \mu_o = 0$, x and y positive simultaneously implies (cf. (20)) that $\lambda(t)$ is constant, i.e., x and y constant (cf. (9) and (10)). But

x positive and constant will sooner or later violate $S(t) \geq 0$ since $S_o < +\infty$. In other words, x and y positive simultaneously is impossible if $\mu(t) = \mu_o = 0$. If $\mu_o > 0$, $\mu(t)$ is increasing, so that (21) implies that R(t) must decrease if $\delta > 0$. If $\delta = 0$ we must have $\dot{R}(t) = 0$, which together with (4) implies x = 0, i.e., x and y positive simultaneously is not possible when $\delta = 0$. We can therefore conclude that the only possibility of having resource extraction and recycling simultaneously is if $\delta > 0$ and $\mu_o > 0$. If this situation ever occurs, it will occur while the stock of harmful residuals is declining.

4. Complete irreversibility.

We shall now consider the case where $\delta = 0$, i.e., where there is no natural depreciation of the harmful residuals in the environment. This is the case which Schulze [18, 1974] has treated, but he disregards the possibility of recycling as an alternative to extraction with environmental effects.

Define $\gamma(t) = \lambda(t) + \mu(t)$. When $\delta = 0$ it follows from (7) and (8) that

$$\dot{\gamma} = r\dot{\gamma} - E'(R) \quad , \tag{22}$$

so that $\dot{\gamma} = 0$ implies $\gamma = E'(R)/r$, which is an increasing function of R when $E'' > 0$.

In section 3 we showed that x and y could not both be positive simultaneously when $\delta = 0$. Together with (4), (9) and (10) this implies that

$$\dot{R} = \begin{cases} f(\gamma) & \text{for } \gamma < c + a\lambda \ , \\ af(c+a\lambda) & \text{" } \gamma > c + a\lambda \ , \end{cases} \tag{23}$$

where

$$f^{-1}(x) \equiv U'(x) \ .$$

Let us first consider the case where $a = 0$, i.e., recycling has no net impact on the environment. The phase diagram for this case is drawn in figure 1. $\gamma(t)$ is increasing, constant or decreasing depending on whether $\gamma - E'(R)/r$ is positive, constant or negative (cf. (21)). $R(t)$ is increasing or constant depending on whether $\gamma - c$ is negative or positive. The initial value of R is given ($= R_o$), and as long as the resource constraint $S(t) \geq 0$ is effective the value of R when the resource

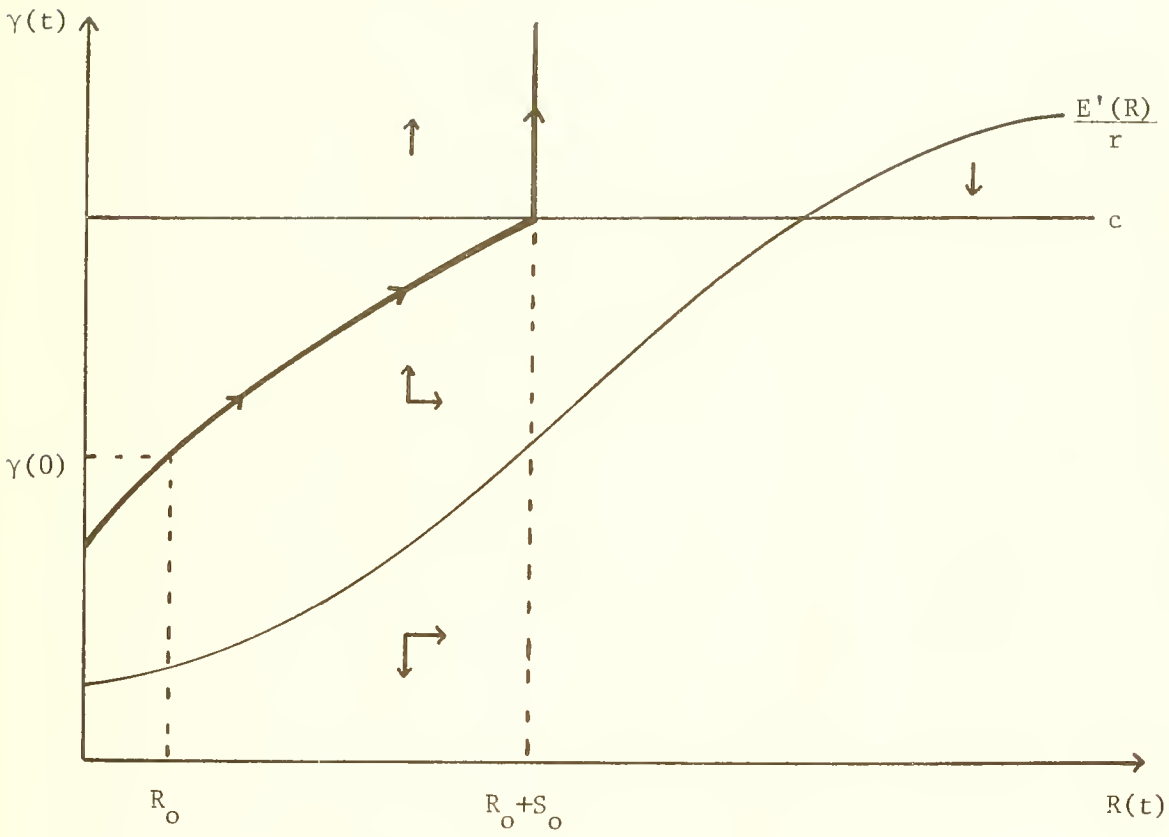

Figure 1

is completely exhausted must be $R_o + S_o$ (cf. (4) and (6)). From (9) and (10) we know that the switch from extraction to recycling occurs when $\gamma(t)$ reaches c. The initial total shadow price of the resource ($= \gamma(0)$) is therefore determined so that $R(t) = R_o + S_o$ and $\gamma(t) = c$ are reached simultaneously.

Before proceeding, let us see what happens if $R_o + S_o$ is so large that $E'(R_o + S_o)/r > c$. In this case the point $R(t) = R_o + S_o$, $\gamma(t) = c$ cannot be reached, and the optimal solution is to leave some of the resource unextracted. The optimal stock of unextracted resource, S*, is determined by $E'(R_o + S_o - S^*)/r = c$, which makes the discounted environmental cost of extracting an additional unit of the resource equal the unit cost of recycling. Formally, this follows from (18), which will hold only if $\gamma(t) = \lambda(t)$ reaches c and $E'(R)/r$ simultaneously.

If the constraint $S(t) \geq 0$ is binding like in figure 1, it is clear that the long-run stock of harmful residuals $R^* = R_o + S_o$ will be unaffected by the recycling cost. We also see from figure 1 that the lower c is, the lower must $\gamma(0)$ be for $\gamma(t)$ to reach c at the same time as R(t) reaches R*. From (4) and (23) we see that this means that the initial resource extraction and the initial growth of harmful residuals is higher the lower is the recycling cost.

If $R^* < R_o + S_o$, like in figure 2, we see that R* is lower the lower the cost of recycling is. Notice that this conclusion differs from the conclusion we had when both δ and a where positive (cf. (15)). Another way of expressing this conclusion is that when it is not optimal to exhaust the resource completely, the optimal total amount of extraction is lower the lower the cost of recycling is. We also see from figure 2 that the lower c is, the lower must $\gamma(0)$ be for $\gamma(t)$ to reach c and

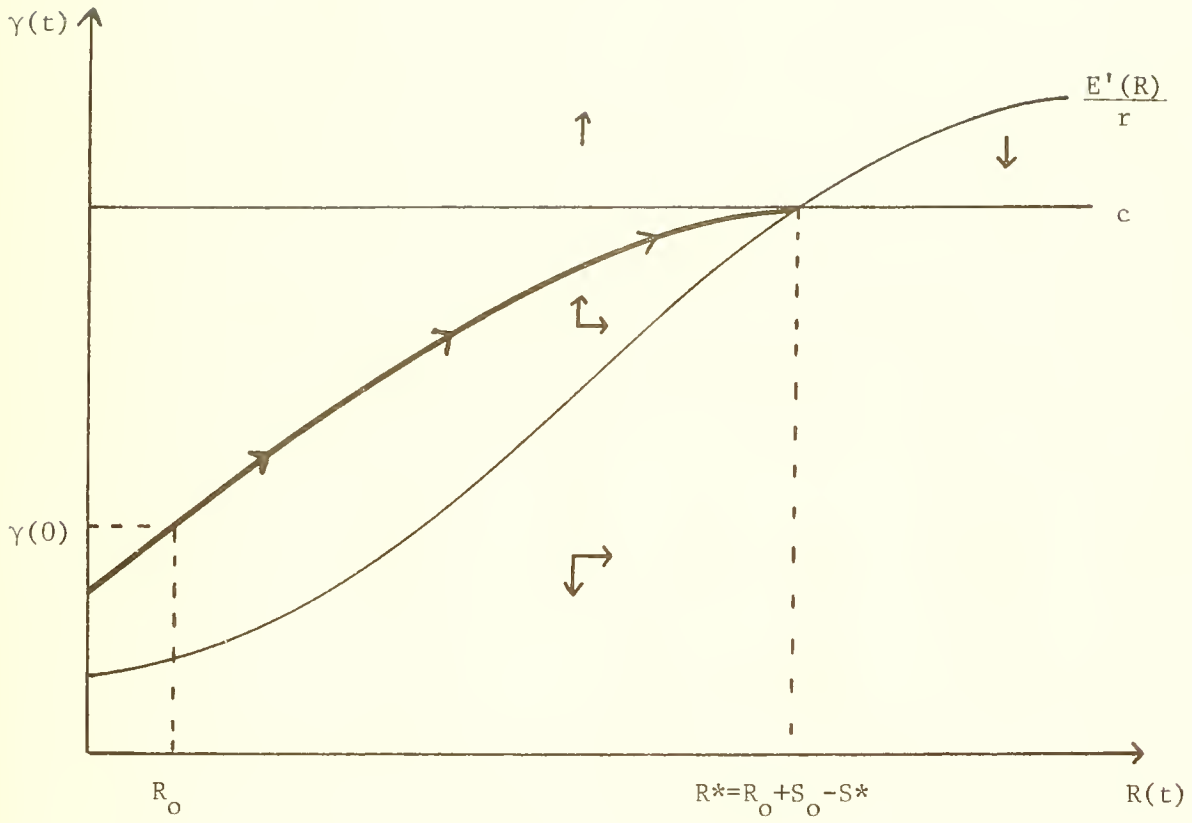

Figure 2

$E'(R)/r$ simultaneously. In other words, also when the resource constraint is not binding we find that the initial resource extraction and initial growth of harmful residuals is higher the lower the cost of harmful residuals is.

Let us now turn to the case where $a > 0$. From (23) we see that in this case $\dot{R} > 0$ also after the switch from extraction to recycling, although $\dot{R}$ at the time of the switch jumps downwards. Let us first look at the development of $\lambda(t)$ and $R(t)$ after the time to when one switches from resource extraction to recycling. Defining $\xi(t) = c + a\lambda(t)$, it follows from (7) (with $\delta = 0$) and (23) (with $\gamma > c + a\lambda$) that

$$\dot{\xi} = r(\xi - c) - aE'(R) \quad , \tag{24}$$

$$\dot{R} = af(\xi) \quad . \tag{25}$$

Together with some initial condition $R(t_o) = \hat{R}$ and (12), which implies

$$\lim_{t \to \infty} e^{-rt}\xi(t) = 0 \quad ,$$

(24) and (25) determine time paths for $\xi(t)$ and $R(t)$. These paths are illustrated by the phase diagram in figure 3. The initial value $\xi(t_o)$ must be such that $\xi(t)$ reaches $U'(0)$ and $c + aE'(R)/r$ at the same time, otherwise (18) will not hold, i.e., $e^{-rt}\xi(t)$ will not approach zero.

If the resource constraint is binding, we must obviously have $\hat{R} = R_o + S_o$. The development of $\gamma(t) = \lambda(t) + \mu(t)$ and $R(t)$ before t_o in this case follows from (22) and (23) (with $\gamma < c + a\lambda$). The initial value $\gamma(0)$ is determined so that $\gamma(t)$ reaches $c + a\lambda(t_o) = \xi(t_o)$ at the same time as $R(t)$ reaches $\hat{R} = R_o + S_o$, cf. the phase diagram in figure 4. This diagram is drawn so that $\xi(t_o) > E'(\hat{R})/r$. This must obviously always hold, since $E'(\hat{R})/r = c + aE'(\hat{R})/r$ and $\xi(t_o) > c + aE'(\hat{R})/r$ from figure 3.

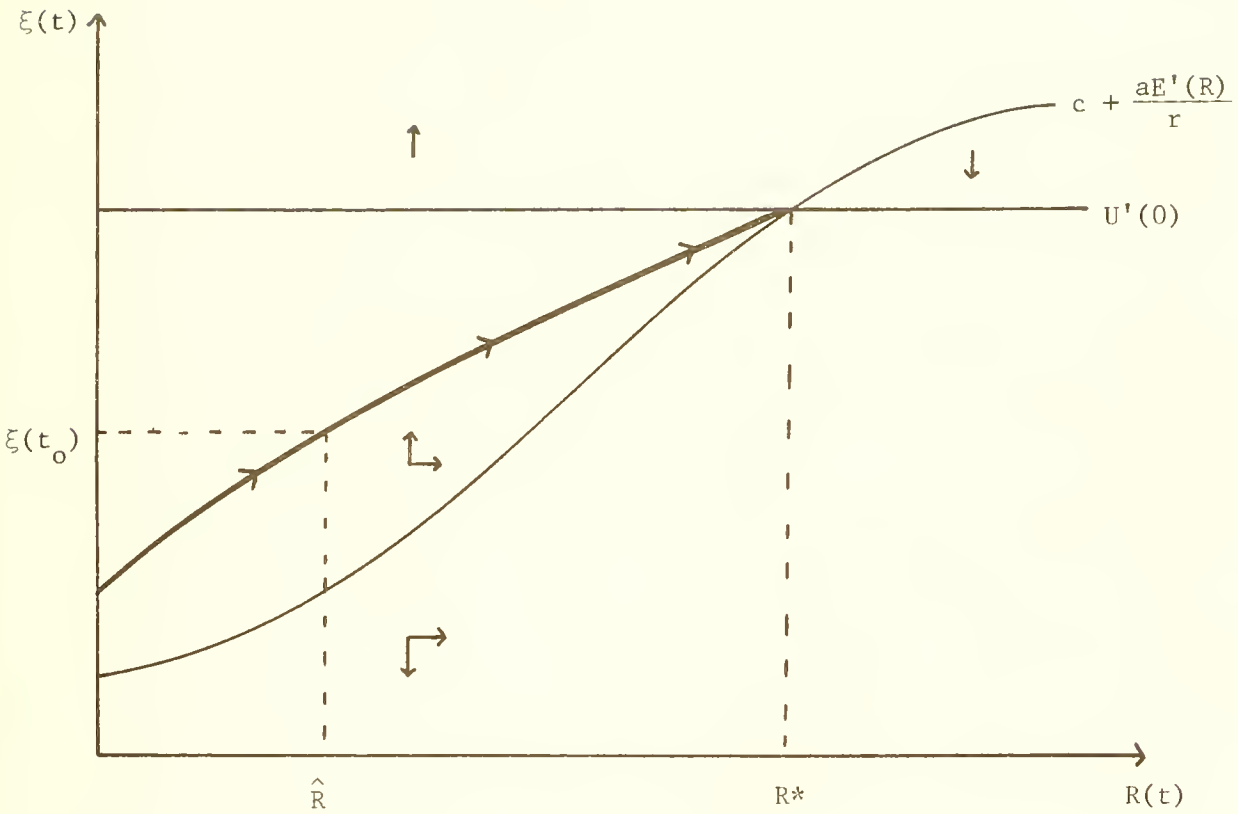

Figure 3

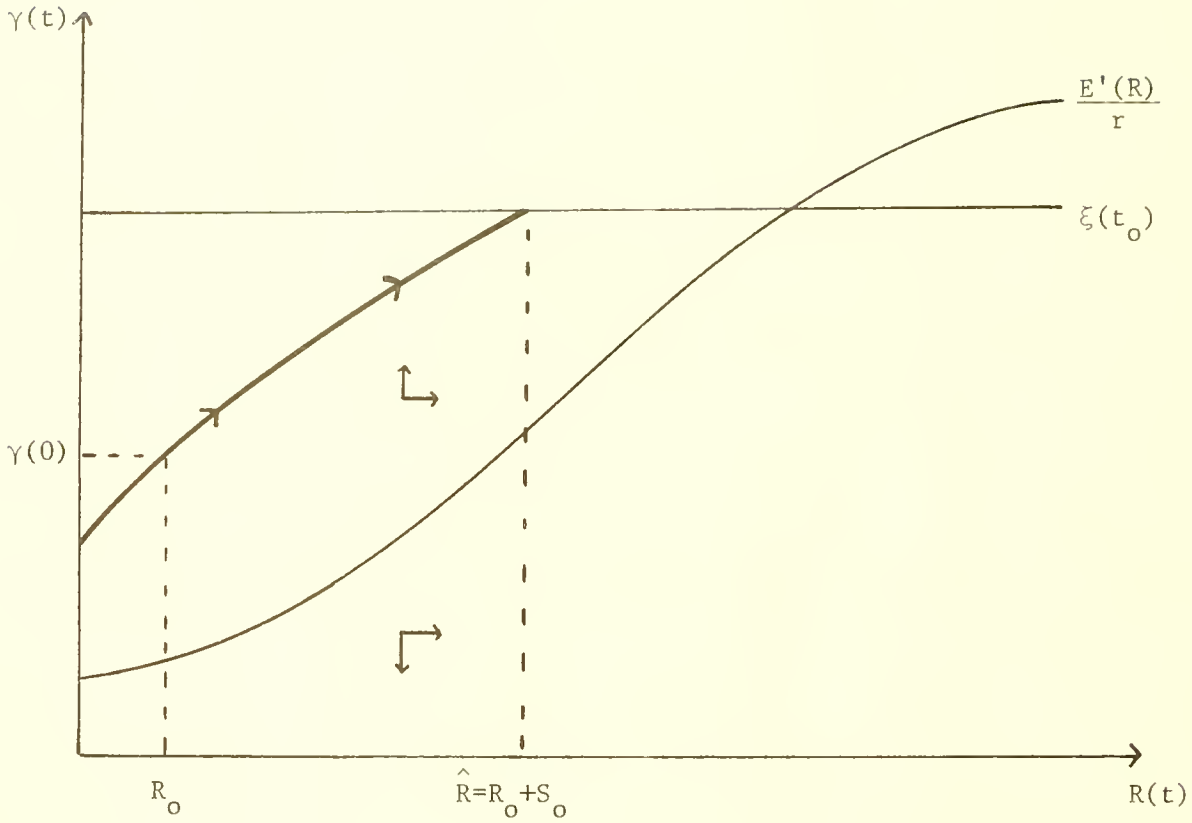

Figure 4

By the definition of $\xi(t_o)$ we have $\xi(t_o) = c + a\lambda(t_o)$, or $\lambda(t_o) = (\xi(t_o) - c)/a$. Together with our analysis above giving $\xi(t_o) = \gamma(t_o) \geq \lambda(t_o)$ (because $\mu(t_o) \geq 0$) this implies

$$\xi(t_o) \leq \frac{c}{1-a} .$$

If the solution just described does not satisfy this inequality, this means that the resource constraint cannot be binding, i.e., $\hat{R} < R_o + S_o$. In this case $\xi(t_o) = c/(1-a)$, and $\hat{R}$ is determined so that $\xi(t_o)$ gives a development of $\xi(t)$ as described by figure 3.

The case where the resource constraint is not binding is perhaps best illustrated in a single figure describing the development of λ and R for all t: When $\mu(t) \equiv 0$, (21) and (22) are simply

$$\dot{\lambda} = r\lambda - E'(R) , \tag{26}$$

$$\dot{R} = \begin{cases} f(\lambda) & \text{for } \lambda < \frac{c}{1-a} , \\ af(c+a\lambda) & \text{" } \lambda > \frac{c}{1-a} . \end{cases} \tag{27}$$

These two differential equations together with $R(0) = R_o$ and (12) determine the development of $\lambda(t)$ and $R(t)$. This is illustrated by the phase diagram in figure 5. The reason why the curve describing this development is kinked at $\lambda = c/(1-a)$ is that $a < 1$, i.e., recycling has less negative impact on the environment than resource extraction has. The initial value $\lambda(0)$ is determined so that $\lambda(t)$ reaches $(U'(0) - c)/a$ and $E'(R)/r$ at the same time. It is easy to see that no other development of $\lambda(t)$ will satisfy (18). From what we have said before, the solution is given by figure 5 only if the value $\hat{R}$ implied by this solution satisfies $\hat{R} \leq R_o + S_o$.

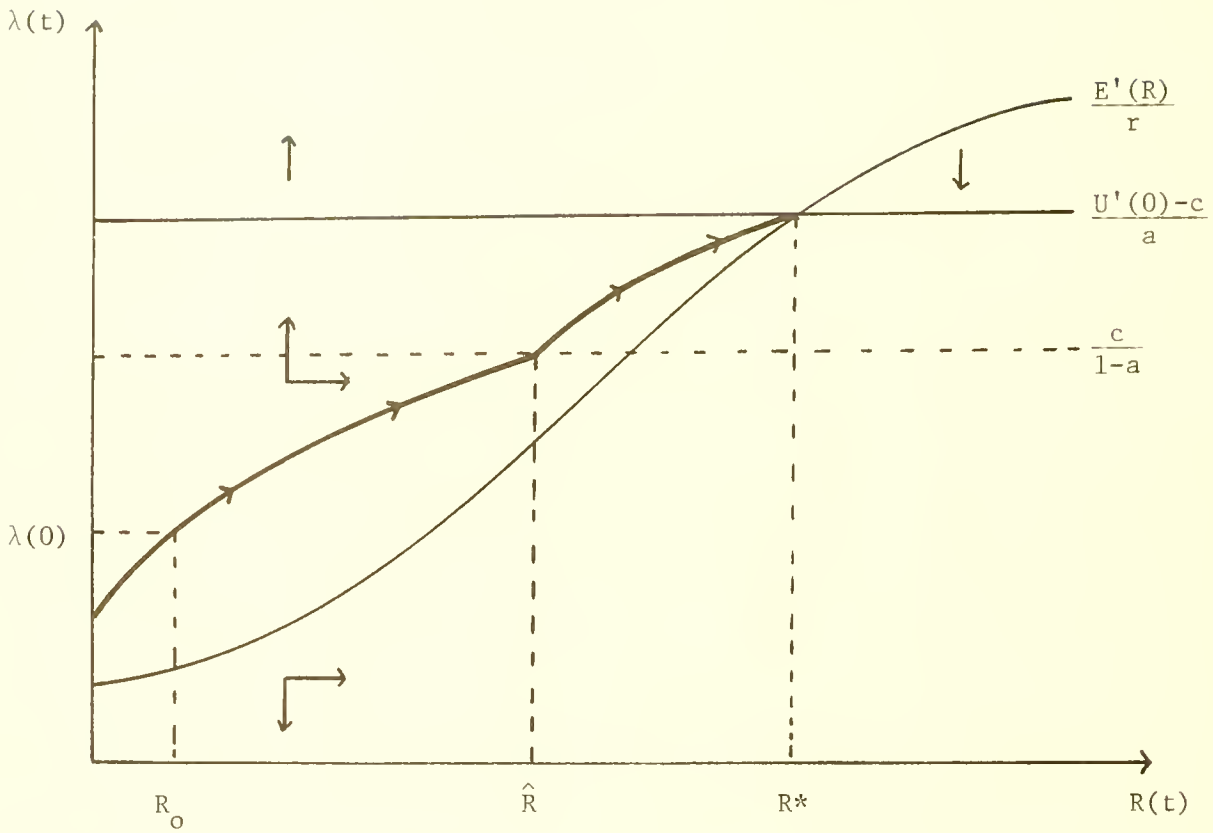

Figure 5

Irrespectively of whether or not the resource constraint is binding, we see that the long-run stock of harmful residuals is determined by $c + aE'(R^*)/r = U'(0)$, which is identical to (13) when $y^* = \delta = 0$. The results given by (15) are still valid, except that $\partial R^*/\partial a \geq 0$ is now excluded. In particular, the long-run stock of harmful residuals will be higher the lower the recycling cost is. Let us now see how the recycling cost affects the initial resource extraction and initial growth of harmful residuals. We first consider the case where the resource constraint is binding. The path describing the development of ξ and R in figure 3 is steeper the lower is c ($\partial(\dot{\xi}/\dot{R})/\partial c < 0$, cf. (24) and (25)), and the curve $c + aE'(R)/r$ lies lower the lower is c. The consequence of a lower c is therefore a lower value of $\xi(t_o)$ (as well as a higher R^*, which we already have pointed out). But from figure 4 it is clear that this means that $\gamma(0)$ must be lower, i.e., the initial resource extraction and initial growth of harmful residuals must be higher, the lower c is. When $\delta = 0$, $a > 0$ and the resource constraint is binding, we therefore have the somewhat surprising conclusion that the lower the cost of recycling is, the higher will the initial resource extraction, the initial growth of harmful residuals and the long-run stock of harmful residuals be.

If $\hat{R} < R_o + S_o$, the effect of c on the initial resource extraction can be analyzed by the help of figure 5. From (26) and (27) we see that $\partial(\dot{\lambda}/\dot{R})/\partial c = 0$ for $\lambda < c/(1-a)$ and $\partial(\dot{\lambda}/\dot{R})/\partial c > 0$ for $\lambda > c/(1-a)$. In other words, the path describing the development of λ and R will be flatter the lower is c for $\lambda > c/(1-a)$, while the steepness of this path will be unaffected by c for $\lambda < c/(1-a)$. We therefore have three effects of lowering c in figure 5: $(U'(0) - c)/a$ increases, $c/(1-a)$ decreases,

and the path from $\lambda = c/1-a$ to $\lambda = (U'(0) - c)/a$ becomes flatter. Let us see in what direction each of these effects moves $\hat{R}$ and $\lambda(0)$. An increase in $(U'(0) - c)/a$ will increase R^*, decrease $\hat{R}$ and increase $\lambda(0)$. A decrease in $c/(1-a)$ will decrease $\hat{R}$ and decrease $\lambda(0)$. Flattening the path from $\lambda = c/(1-a)$ to $\lambda = (U'(0) - c)/a$ will decrease $\hat{R}$ and increase $\lambda(0)$. We can therefore conclude that in addition to increasing R^*, a lower recycling cost will imply a lower $\hat{R}$, i.e., a lower total resource extraction. The effect of a lower recycling cost on $\lambda(0)$, and therefore on initial extraction and initial growth of harmful residuals, will depend on the functions $U(x)$ and $E(R)$.

In all our reasoning we have assumed that some resource extraction takes place before recycling starts. This will be true as long as $R_o < \hat{R}$, with $\hat{R}$ determined like in figure 2 (for $a = 0$) or figure 5 (for $a > 0$). It is easy to see from these figures that our condition $E'(R_o)/(r+\delta) < c/(1-a)$ is necessary for any resource extraction to occur. When $a = 0$ this condition is also sufficient, but when $a > 0$ it follows from figure 5 that we can have $R_o > \hat{R}$ even if the inequality above holds. The condition $E'(R_o)/(r+\delta) < c/(1-a)$ together with $U'(0) > c/(1-a)$ will always be sufficient to make it optimal to have some recycling. The situation where it is optimal to leave the entire resource stock unextracted is not particularly interesting. However, one result for this case should be mentioned: From our reasoning of how c affects $\lambda(0)$ when $a > 0$ (pp. 18-19), it is easy to see that when $a > 0$ and no resource extraction takes place, the initial growth of harmful residuals will be lower the lower the recycling cost is.

We have seen that both for $a = 0$ and $a > 0$ it can be optimal to extract some, but not all, of the available resource stock. This conclusion

holds also when the possibility of recycling is excluded, cf. Schulze [18, 1974]. Schulze also finds the possibility of having a positive resource extraction after a period with no extraction. It is easy to verify from our phase diagrams that such a situation can never occur in the present model. The reason for this difference compared with Schulze is not that we have included recycling, but that Schulze assumes a fixed, finite horizon. As one comes close to this horizon, the shadow price of an additional unit of harmful residuals declines, since the disutility of an increased stock of residuals will only be integrated over a short period. It is therefore possible to find it optimal to have resource extraction towards the end of the planning period although this was not optimal earlier. With an infinite horizon, like we have assumed, this possibility is excluded.

5. Nonbinding resource constraint.

We shall now consider the case where $\delta > 0$, but where the constraint $S(t) \geq 0$ is not binding. In this case the differential equations determining λ and R follow from (4), (7), (9) and (10):

$$\dot{\lambda} = (r+\delta)\lambda - E'(R) \quad , \tag{28}$$

$$\dot{R} = \begin{cases} f(\lambda) - \delta R & \text{for } \lambda < \frac{c}{1-a} \quad , \\ af(c+a\lambda) - \delta R & \text{" } \lambda > \frac{c}{1-a} \quad . \end{cases} \tag{29}$$

Together with the initial value R_o and the condition (12) these two differential equations determine the paths for λ and R, provided that the corresponding solution for x does not violate $S(t) \geq 0$. The solution is illustrated in the phase diagram in figure 6. The curve giving $\dot{\lambda} = 0$ is as usual given by $\lambda = E'(R)/(r+\delta)$, cf. (28). The curve giving $\dot{R} = 0$ follows from (29), and is discontinuous at $\lambda = c/(1-a)$ since $a < 1$. As "a" approaches zero, the upper part of this broken curve approaches the vertical axis. The heavily drawn curve marked with arrows is the only development of λ and R which satisfies (18). This curve is kinked at $\hat{R}$ (= the stock of harmful residuals at the time of the switch from resource extraction to recycling) because $a < 1$.

The long-run stock of harmful residuals is given from figure 6 by $U'(\delta R^*/a) = c + aE'(R^*)/(r+\delta)$, which of course is identical to equation (13). Notice that figure 6 is drawn so that the curves giving $\dot{\lambda} = 0$ and $\dot{R} = 0$ intersect at a value $\lambda^* > c/(1-a)$. If the curve for $\dot{\lambda} = 0$ had lied to the right of the upper part of the curve for $\dot{R} = 0$ for all $\lambda \geq c/(1-a)$, the long-run value R^* would have implied a long-run positive

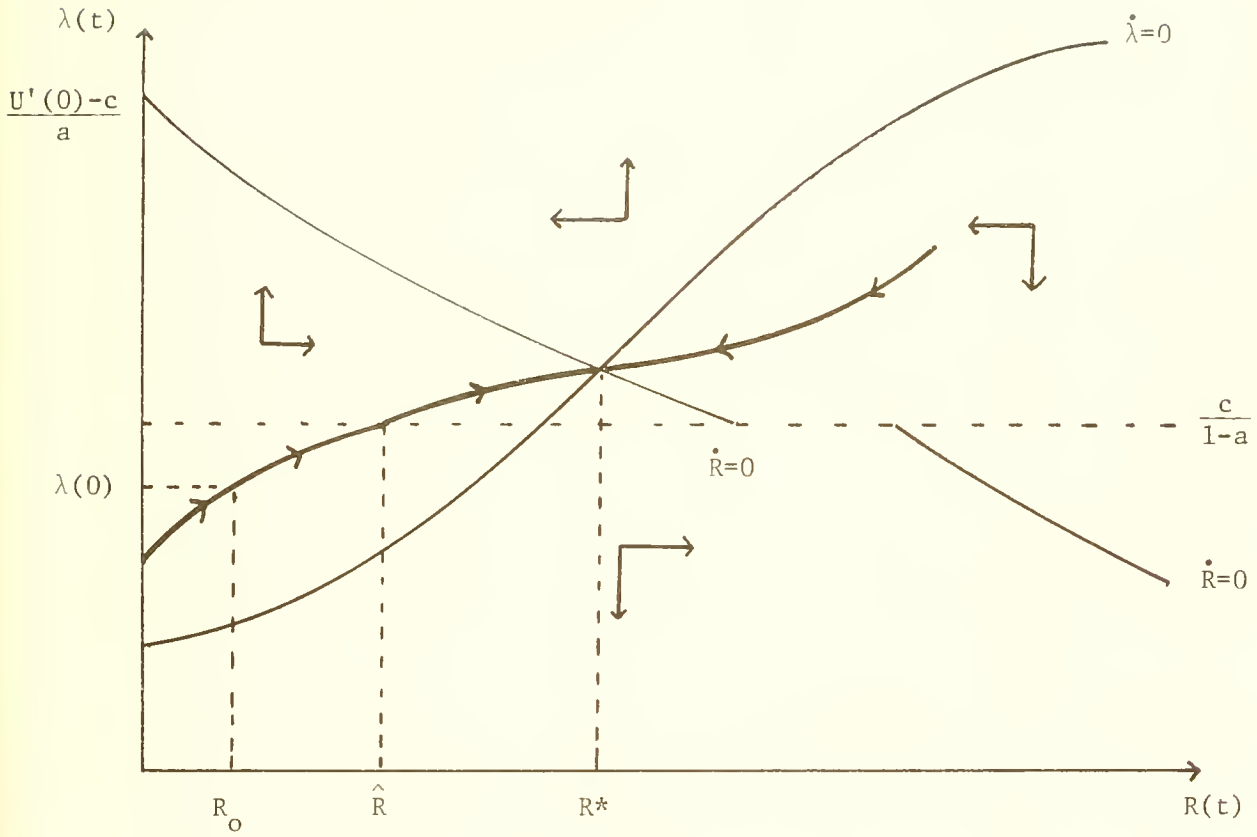

Figure 6

value of resource extraction. Since this sooner or later would violate the constraint $S(t) \geqq 0$, we can conclude that a necessary condition for the solution to follow from figure 6 is that $\lambda^* = E'(R^*)/(r+\delta) \geqq c/(1-a)$, where R^* follows from (13). If $a = 0$ when $\delta > 0$, $R(t)$ will approach zero, so that the necessary condition in this case becomes $E'(0)/(r+\delta) \geqq c$, which will imply that no resource extraction should ever take place. When $a > 0$, the solution following from figure 6 may or may not imply some resource extraction. Notice that the condition $E'(R_o)/(r+\delta) < c/(1-a)$ is necessary, but not sufficient for resource extraction to take place. If the solution does give $\lambda(0) < c/(1-a)$, implying some resource extraction, the solution will of course only be valid if the constraint $S(t) \geqq 0$ is not violated.

Let us now see how R^*, $\hat{R}$ and $\lambda(0)$ will be affected by the cost of recycling. If $a = 0$ our solution will be $U'(y^*) = c$, $\hat{R} = R_o$ and $R(t) \to 0$ whatever value c has. If $a > 0$, a reduction of c will decrease $c/(1-a)$, shift $(U'-c)/a$ to the right and make the path leading from $\lambda = c/(1-a)$ to $\lambda = \lambda^*$ flatter (cf. (28) and (29), which imply $\partial(\dot{\lambda}/\dot{R})/\partial c > 0$ for $\lambda > c/1-a)$. Decreasing $c/(1-a)$ will decrease $\hat{R}$ and decrease $\lambda(0)$ (for $\lambda(0) < c/(1-a)$). Shifting $(U'-c)/a$ to the right will increase R^*, decrease $\hat{R}$ (for $\lambda(0) < c/(1-a)$) and increase $\lambda(0)$. Flattening the path leading from $\lambda = c/(1-a)$ to $\lambda = \lambda^*$ will decrease $\hat{R}$ (for $\lambda(0) < c/(1-a)$) and increase $\lambda(0)$. We can therefore draw the following conclusion: The lower the cost of recycling is, the higher is the long-run stock of harmful residuals and the lower is this stock at the time when one switches from resource extraction to recycling, provided that some extraction takes place. If no resource extraction takes place, $\lambda(0)$ will be higher, i.e., the initial growth of harmful residuals will be lower, the lower

is the cost of recycling. When some resource extraction takes place, the initial extraction and initial growth of harmful residuals will be lower, unaffected or higher the lower the recycling cost is, depending on the functions U(x) and E(R).

6. Constant marginal environmental cost.

In this section we shall consider the case where $E(R) = bR$, i.e., $E'(R) = b$, where b is some positive constant. In this case it follows from (16) that the shadow price of an additional unit of harmful residuals is

$$\lambda(t) = \frac{b}{r+\delta} \quad , \tag{30}$$

i.e., constant. Inserting this constant marginal environmental cost into (21) gives us

$$(r+\delta)c - \delta\mu(t) - (1-a)b = 0 \quad ,$$

which can only be satisfied at one time point (for $\mu(t)$ positive and growing). In other words, resource extraction and recycling cannot take place simultaneously in the present case.

When recycling takes place, it is constant and equal to y*, cf. (10) and (30):

$$U'(y^*) = c + \frac{ab}{r+\delta} \quad , \tag{31}$$

which gives $y^* > 0$ because of our assumption $b/(r+\delta) = E'(R_o)/(r+\delta) < c/(1-a) < U'(0)$ and $U'' < 0$.

Resource extraction will take place whenever $\lambda + \mu = b/(r+\delta) + \mu$ is smaller than $U'(y^*)$, i.e., whenever $\mu(t) < c - (1-a)b/(r+\delta)$ (cf. (9) and (10)). Notice that our assumption $b/(r+\delta) < c/(1-a)$ implies that $\mu_o > 0$, otherwise we would get a positive and constant resource extraction forever, violating $S(t) \geq 0$. If the inequality above was not satisfied, it would be optimal to leave the entire stock unextracted. In other words, in the present case either all or none of the resource is extracted.[8)]

From (19) and (30) we find that when resource extraction takes place we have

$$(32) \qquad \frac{d}{dt} U'(x) = r[U'(x) - \frac{b}{r+\delta}] \ ,$$

which is a standard result in the theory of resource extraction, except that $b/(r+\delta)$ now represents environmental costs instead of traditional extraction costs. The initial resource extraction $x(0)$ is determined so that the resource is completely exhausted at the same time as $U'(x)$ reaches the total cost of recycling. The time path of $U'(x+y)$ is illustrated in figure 7.

Let us see how the recycling cost affects the resource extraction in the case where $S(t_o) = 0$, i.e., all of the resource is extracted. From (32) and figure 6 it is easy to see that a lower c must give a lower $U'(x(0))$, and therefore a higher $x(0)$, due to the fact that the entire resource stock is assumed to be extracted for both values of c. In other words, our conclusion about a lower recycling cost giving a higher initial extraction and higher initial growth of harmful residuals holds also in the present case.

We may also be interested in how the evaluation of the environmental effects affects the resource extraction. From (32) we see that the lower b is, the steeper is the curve for $U'(x)$ in figure 6 (for any given x). For a given value of $c + ab/(r+\delta)$, this would give a lower initial value of $U'(x(0))$ and a lower t_o. If $a > 0$, a lower value of b will also mean a lower value of $c + ab/(r+\delta)$, which also has the effect of reducing $U'(x(0))$. We can therefore conclude that the initial extraction and growth of harmful residuals is higher the less harmful the environmental effects are regarded.

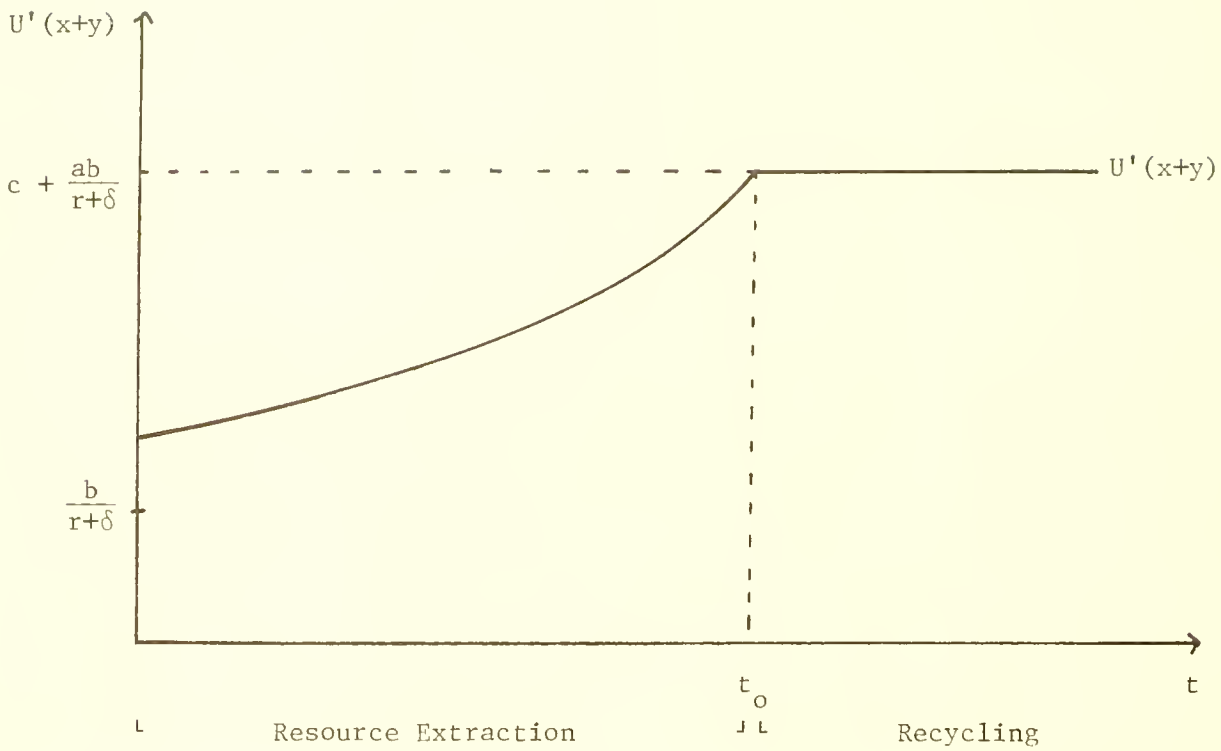

Figure 7

So far, we have said nothing about the optimal development of harmful residuals, except how this stock grows initially. From (4) it is clear that if $\delta = 0$ we must get $R^* = R_o + S_o$ (disregarding the case where it is optimal not to extract the resource) if $a = 0$ and $\dot{R} = ay^* > 0$, i.e., $R(t) \to \infty$, if $a > 0$. The assumption about a constant marginal environmental cost is at best an approximation to some situations. Such an approximation may not be too wrong over certain regions of R, but will clearly be misleading when $R \to \infty$. The approximation $E'(R) = b$ is therefore not suited to treat the case with $\delta = 0$, $a > 0$.

When $\delta > 0$, we know that (cf. (4))

$$\dot{R} = [x + ay] - \delta R \ .$$

If our previous results for $x(t)$ and $y(t)$ as well as $R(0) = R_o$ are inserted into this differential equation, it can be solved to find an explicit solution for $R(t)$:

$$R(t) = e^{-\delta t}\left[R_o + \int_0^t [x(\tau) + ay(\tau)]e^{\delta\tau}d\tau\right] \ .$$

There are several possible developments of $R(t)$. We may have $R(t)$ increasing as long as resource extraction takes place. However, it is also possible that as $x(t)$ declines and $R(t)$ grows, we reach a time point when $R(t)$ starts to decline, although resource extraction still is taking place. Once the resource is completely extracted, $R(t)$ will fall towards zero if $a = 0$. If $a > 0$, $R(t)$ may rise or decline at first, depending on a, δ, y^* and the magnitude of $R(t)$ at the time recycling starts. In any case, $R(t)$ will approach $R^* = ay^*/\delta$, where y^* is determined by (31). It can easily be verified that R^* and y^* in this case depends on c, a, δ and r in the same way as in the general case treated in section 3

(cf. (15)). Furthermore, since $\partial y^*/\partial b < 0$ follows from (31), we must have $\partial R^*/\partial b < 0$. In other words, the less harmful the environmental effects are regarded, the higher is the optimal long-run stock of harmful residuals.

7. Conclusions.

In this section we shall repeat some of the conclusions we found for the cases studied. In these cases resource extraction and recycling will never take place simultaneously. At first only resource extraction takes place, and the optimal rate of extraction declines over time. When recycling starts, it is either constant (for $a = 0$) or declining (for $a > 0$). In the latter case the rate of recycling approaches a stationary level, which will be positive if and only if $\delta > 0$ (i.e., some natural degrading of harmful residuals).

The existence of environmental effects may make it optimal to leave some of a finite resource stock unextracted. Even when it is optimal to extract the entire resource stock, in spite of the environmental consequences, the optimal path of extraction will be affected by the existence of environmental effects.

The stock of harmful residuals, both in the long run (R^*) and at the time when resource extraction stops ($\hat{R}$) will usually be affected by the recycling costs (c). The same holds for the initial growth of harmful residuals, which is determined by the initial resource extraction ($x(0)$). Tables I-III summarize how c affects R^*, $\hat{R}$ and $x(0)$ for the cases we have treated; cases not treated are crossed out. The tables do not treat cases where no resource extraction takes place. From table II we see that whenever $\hat{R}$ is affected by c, this stock is lower the lower is c. The opposite conclusion holds for R^*, except for the case where $\delta = a = 0$, when $R^* = \hat{R}$ (see table I). From table III we see that for most of our cases, the initial resource extraction and growth of harmful residuals will be higher the lower is the recycling cost. Although this may seem

Table over $\partial R^*/\partial c$:

	$S(t) \geqq 0$ nonbinding	$S(t) \geqq 0$ binding, $E'' > 0$	$S(t) \geqq 0$ binding, $E'' = 0$
$\delta = 0$ $a = 0$	positive	zero	zero
$\delta = 0$ $a > 0$	negative	negative	$R(t) \to +\infty$
$\delta > 0$ $a = 0$	$R(t) \to 0$	$R(t) \to 0$	$R(t) \to 0$
$\delta > 0$ $a > 0$	negative	negative	negative

Table I

Table over $\partial\hat{R}/\partial c$:

	$S(t) \geqq 0$ nonbinding	$S(t) \geqq 0$ binding, $E'' > 0$	$S(t) \geqq 0$ binding, $E'' = 0$
$\delta = 0$ $a = 0$	positive	zero	zero
$\delta = 0$ $a > 0$	positive	zero	zero
$\delta > 0$ $a = 0$	no resource extraction		
$\delta > 0$ $a > 0$	positive		

Table II

Table over $\partial x(0)/\partial c$:

	$S(t) \geqq 0$ nonbinding	$S(t) \geqq 0$ binding, $E'' > 0$	$S(t) \geqq 0$ binding, $E'' = 0$
$\delta = 0$ $a = 0$	negative	negative	negative
$\delta = 0$ $a > 0$	undetermined	negative	negative
$\delta > 0$ $a = 0$	no resource extraction		negative
$\delta > 0$ $a > 0$	undetermined		negative

Table III

somewhat surprising, it corresponds to the usual result that initial extraction is higher the lower the cost of producing a substitute is (see Hoel [6, 1977], where it is also shown that this result does not necessarily hold when a monopolist owns the resource). Notice, however, that when $a > 0$ this result does not always hold. In the case of $a > 0$ we may get a lower initial extraction and growth of harmful residuals the lower the cost of recycling is.

We have said nothing about how a competitive economy will perform compared with our optimal solution. In the absence of environmental effects, it is well known that a competitive economy under certain conditions will give optimal rates of extraction and recycling (see for instance Weinstein and Zeckhauser [22, 1974]). Even under these conditions the presence of the external environmental effects will in general imply that the competitive solution is nonoptimal. However, from (9) and (10) it can be seen that the competitive solution can be made optimal by introducing Pigouvian taxes $\lambda(t)$ and $a\lambda(t)$ on resource extraction and recycling, respectively, where $\lambda(t)$ is given by (16).

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Footnotes:

1) It is of course a drastic simplification to only treat two types of residuals.

2) See for instance Ayres and Kneese [3, 1969], Kneese, Ayres and d'Arge [10, 1972] and Mäler [13, 1974, ch. 2] for detailed expositions of the material balance approach to environmental economics.

3) Equation (1) disregards the stock of material embodied in durable final goods.

4) Obviously, it would be better to specify two different residuals in this case.

5) Alternatively, we could let the cost of extraction and recycling be c_1 and c_2, respectively, and the gross utility function of using x+y be $U_1(x+y)$. We would then get (5) be defining $U(x+y) = U_1(x+y) - c_1x$ and $c = c_2 - c_1$.

6) For notational convenience, we have not introduced any additional superscript or similar to denote optimal values of R, S, x and y. This should not cause any misunderstanding.

7) Strictly speaking, a transversality condition of the type (12) is in general not necessary for an optimal solution. However, it is easy to show that a solution derived by using (12) must be optimal: First, it is obvious from economic considerations that when $aE'(R)/(r+\delta) > U'(0)$ for R sufficiently large, the solution to our optimization problem is not changed by introducing an additional constraint

$R(t) \leqq N$, where N is a large (but finite) given number, since it cannot be optimal to let $R \to \infty$ with our assumptions. Since our Hamiltonian is concave, a solution $(\tilde{S},\tilde{R},\tilde{x},\tilde{y})$ must be optimal if $\lim_{t\to\infty} \{e^{-rt}\mu(S-\tilde{S}) - e^{-rt}\lambda(R-\tilde{R})\} \geqq 0$ for all feasible paths $\delta(t)$ and $R(t)$ (cf. Seierstad and Sydsaeter [19, 1977, theorem 10]). But $\mu \geqq 0$, $S \geqq 0$ and $\lim_{t\to\infty} \mu\tilde{S} = 0$, (12) therefore implies that this inequality holds since $R(t) \leqq N$.

8) This is true as long as the special case $b/(r+\delta) = c/(1-a)$ is disregarded. In this case it makes no difference to society how much of the resource stock is extracted.

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